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A phenomenological description of $K \to \pi\pi\gamma$ magnetic transitions*

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Abstract: A phenomenological analysis of $K \to \pi\pi\gamma$ ($K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$) with the direct emission photon is carried out beyond the leading order in the chiral perturbation theory. We show that the experimental evidence for the large photon energy dependence of the magnetic amplitude in $K_L \to \pi^+\pi^-\gamma$ seems to indicate an interesting consequence: vector meson dominance must be implemented at $O(p^4)$, which is not a general feature of the chiral perturbation theory. The phenomenology of $K^+ \to \pi^+\pi^0\gamma$ is also analyzed using the same scheme.

Keywords: Kaon Physics, Rare Decays, Chiral Lagrangians.

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Contents

1. Introduction

Non-leptonic kaon decays have been an important tool for studying the weak interactions [1]–[5]. The radiative non-leptonic kaon decays such as the processes $K \to \pi\pi\gamma$ ($K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$) are dominated by long distance contributions, and it is not easy to match long and short distance contributions of these processes. In ref. [2], an impressive improvement for the evaluation of the weak matrix elements in some particular processes matching long and short distance contributions is obtained in the large N_C limit. Spin-1 resonances are implemented in this context and important to achieve the matching. Here we somewhat complement their work by looking for a good low energy phenomenological description of the spin-1 resonances in the weak sector to study the decays of $K \to \pi\pi\gamma$.

The total amplitude of $K \to \pi\pi\gamma$ contains two kinds of contributions: the inner bremsstrahlung (IB) and direct emission (DE). Due to the pole in the photon energy the IB amplitude generally dominates unless the non-radiative one is suppressed due to some particular reason. This is the case of $K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$. The non-radiative amplitude of the former is suppressed by CP invariance and the latter one is suppressed due to the $\Delta I = 1/2$ rule. It is of interest to extract the DE amplitude of these channels in order to reveal the chiral structure of the processes. DE contribution can be decomposed into electric and magnetic parts in a multipole expansion [6, 7]. The available experimental evidence is consistent with a dominant magnetic part for the DE amplitude [8, 9]. So we will focus our attention on the magnetic part.

In the framework of chiral perturbation theory(χPT) [10, 11], $K \to \pi\pi\gamma$ has been analyzed previously [4, 6, 7], [12]–[16], and the leading order of the magnetic amplitudes of the processes, starting at $O(p^4)$, appear as a constant with two kinds of contributions: (i) the reducible type from Wess-Zumino-Witten action and $O(p^2)$ weak lagrangian $\mathcal{L}_2^{\Delta S=1}$; (ii) the local type from $O(p^4)$ weak lagrangian $\mathcal{L}_4^{\Delta S=1}$. However, experimental analysis has found a clear and large dependence on the photon energy in $K_L \to \pi^+\pi^-\gamma$ [9, 17]. In the K^+ case, the energy dependence of the magnetic amplitude has not been observed yet. In order to explain the photon energy dependence in χPT, the theoretical analysis has to be beyond the leading order [4, 15, 16]. A complete $O(p^6)$ magnetic amplitude of $K \to \pi\pi\gamma$ generally may be useful in the future but not now since some unknown parameters have to be introduced, which in fact makes the prediction impossible.

The recent direct measurement of the $K_L \to \pi^+\pi^-\gamma$ DE form-factor by KTeV Collaboration [17] clearly indicates a vector meson dominance (VMD) form-factor

$$\mathcal{F} = \frac{A_1}{1 - \frac{m_K^2}{m_V^2} + \frac{2m_K}{m_V^2} E_\gamma^*} + A_2 \,, \tag{1.1}$$

where A_1 and A_2 are constants with $A_1/A_2 = -1.243 \pm 0.057$, and E_γ^* is the photon energy in the K_L rest frame. Equation (1.1) gives the best χ^2 for a single-parameter fit (χ^2/DOF is 38.8/27), compared with the linear slope fit (χ^2/DOF is 43.2/27), and two-parameter quadratic slopes fit (χ^2/DOF is 37.6/26). Therefore, this measurement seems to indicate that VMD should be implemented at $O(p^4)$ instead of being at $O(p^6)$ for this decay. This point is not well understood currently within χPT. The purpose of the present paper is to understand it using a phenomenological description. Also, we extend our analysis to the decay $K^+ \to \pi^+\pi^0\gamma$.

In section 2, we remind briefly the kinematics of $K \to \pi\pi\gamma$. In section 3, we carry out a phenomenological analysis of $K \to \pi\pi\gamma$ using chiral lagrangian plus VMD. The results are summarized in section 4.

2. Kinematics

The general invariant amplitude of $K \to \pi\pi\gamma$ can be defined as follows [4, 13]

$$A\Big[K(p) \to \pi_1(p_1)\pi_2(p_2)\gamma(q,\epsilon)\Big] = \epsilon^\mu(q) M_\mu(q, p_1, p_2) \,, \tag{2.1}$$

where $\epsilon_\mu(q)$ is the photon polarization and M_μ is decomposed into an electric E and a magnetic M amplitudes as

$$M_\mu = \frac{E(z_i)}{m_K^3}\left[p_1{\cdot}q p_{2\mu} - p_2{\cdot}q p_{1\mu}\right] + \frac{M(z_i)}{m_K^3}\epsilon_{\mu\nu\alpha\beta}p_1^\nu p_2^\alpha q^\beta \,, \tag{2.2}$$

with

$$z_i = \frac{q{\cdot}p_i}{m_K^2}\,, \quad (i = 1,2)\,, \qquad z_3 = \frac{p{\cdot}q}{m_K^2}\,, \qquad z_3 = z_1 + z_2 \,.$$

The double differential rate for the unpolarized photon is

$$\frac{\partial^2\Gamma}{\partial z_1 \partial z_2} = \frac{m_k}{(4\pi)^3}\left(|E(z_i)|^2 + |M(z_i)|^2\right)\left[z_1 z_2(1-2z_3-r_1^2-r_2^2) - r_1^2 z_2^2 - r_2^2 z_1^2\right], \quad (2.3)$$

where $r_i = m_{\pi_i}/m_K$.

In $K_L \to \pi^+\pi^-\gamma$, the most useful variables are: (i) the photon energy in the kaon rest frame E_γ^*, and (ii) the angle θ between the photon and π^+ momenta in the di-pion rest frame. The relations between E_γ^*, θ and the z_i are:

$$z_3 = \frac{E_\gamma^*}{m_K}, \qquad z_\pm = \frac{E_\gamma^*}{2m_K}(1 \mp \beta\cos\theta), \quad (2.4)$$

where $\beta = \sqrt{1 - 4m_\pi^2/(m_K^2 - 2m_K E_\gamma^*)}$. Then the differential rate is

$$\frac{\partial^2\Gamma}{\partial E_\gamma^* \partial\cos\theta} = \frac{(E_\gamma^*)^3\beta^3}{512\pi^3 m_K^3}\left(1 - \frac{2E_\gamma^*}{m_K}\right)\sin^2\theta(|E|^2 + |M|^2). \quad (2.5)$$

For $K^+ \to \pi^+\pi^0\gamma$, three photons will be detected in the measurement, so it is more useful to study the differential rate as a function of: (i) the charged pion kinetic energy in the K^+ rest frame T_c^*, and (ii) $W^2 = (q{\cdot}p_K)(q \cdot p_+)/(m_{\pi^+}^2 m_K^2)$ [8]. These two variables are related to the z_i by

$$z_0 = \frac{1}{2m_K^2}\left(m_K^2 + m_{\pi^+}^2 - m_{\pi^0}^2 - 2m_K m_{\pi^+} - 2m_K T_c^*\right), \quad (2.6)$$

$$z_3 z_+ = \frac{m_{\pi^+}^2}{m_K^2} W^2. \quad (2.7)$$

The advantage of using these variables is that, through the W^2 dependence, one can easily disentangle the different contributions of the IB, DE amplitudes, and interference term between IB and DE

$$\begin{aligned}\frac{\partial^2\Gamma}{\partial T_c^*\partial W^2} = \frac{\partial^2\Gamma_{\text{IB}}}{\partial T_c^*\partial W^2}\Bigg[1 &+ \frac{m_{\pi^+}^2}{m_K} 2Re\left(\frac{E_{\text{DE}}}{eA}\right)W^2 + \\ &+ \frac{m_{\pi^+}^4}{m_K^2}\left(\left|\frac{E_{\text{DE}}}{eA}\right|^2 + \left|\frac{M_{\text{DE}}}{eA}\right|^2\right)W^4\Bigg],\end{aligned} \quad (2.8)$$

where $A = A(K^+ \to \pi^+\pi^0)$.

3. Analysis

The leading order magnetic amplitudes of $K \to \pi\pi\gamma$ start at $O(p^4)$ in χPT [4, 15]

$$M_L^{(4)} = \frac{eG_8 m_K^3}{2\pi^2 F}(a_2 + 2a_4), \quad (3.1)$$

$$M_+^{(4)} = -\frac{eG_8 m_K^3}{4\pi^2 F}[2 + 3(2a_3 - a_2)]. \quad (3.2)$$

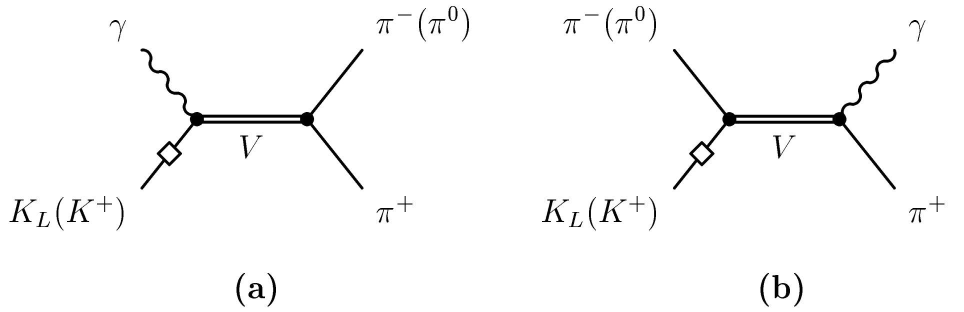

Figure 1: Diagrams contributing to the indirect VMD magnetic amplitude of $K_L \to \pi^+\pi^-\gamma$ or $K^+ \to \pi^+\pi^0\gamma$. The diamond in the external legs denotes $K_L - \pi^0, \eta_8$ or $K^+ - \pi^+$ mixing. The black circle denotes the strong/electromagnetic vertex. The crossed diagram $\pi^+ \leftrightarrow \pi^-(\pi^0)$ should be considered in (b).

The subscripts L and $+$ denote $K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$, respectively. The a_i's parts of the above amplitudes come from the local weak lagrangian $\mathcal{L}_4^{\Delta S=1}$. The first term in $M_+^{(4)}$ is the reducible type contribution. Due to the Gell-Mann-Okubo mass relation, the reducible magnetic amplitude of $K_L \to \pi^+\pi^-\gamma$ generated from $K_L - \pi^0(\eta_8)$ mixing vanishes at $O(p^4)$. However, when η' is included, thus $\eta - \eta'$ mixing is considered, there is a reducible amplitude called F_1 term in refs. [4, 16], which is therefore at $O(p^6)$.

The experimental analysis of $K_L \to \pi^+\pi^-\gamma$ using the VMD form-factor parameterization by KTeV Collaboration [17] indicates that VMD must be implemented at $O(p^4)$. This means that the couplings a_i's, i=1, 2, 3, 4 (or in terms of N_i's, i=28, 29, 30, 31) should get the contribution from the vector resonance exchange.

VMD can be introduced into the effective lagrangian phenomenologically, and there are two different kinds of vector resonance exchange contributions to the nonleptonic radiative kaon decays [18, 19]:

(i) Vector resonance exchange between strong/electromagnetic vertices with a weak transition in an external leg, as shown in figure 1, which are usually called as indirect transitions. The amplitude from this kind of transition is of reducible type, and vanishes at $O(p^4)$, which therefore contributes first at $O(p^6)$.

(ii) The direct weak transitions are those where the weak vertices involving the vector resonances are present, which could contribute to the couplings a_i's. But this is NOT a general feature of VMD. Indeed the antisymmetric tensor realization of vector resonances does not generate any vector exchange contributions to $K \to \pi\pi\gamma$ at $O(p^4)$. However, this is not the case for the other realization approaches such as massive Yang-Mills, hidden local symmetry, and conventional vector formulations. It has been pointed out in ref. [18] that, for the odd-intrinsic parity operator relevant in the $V \to P\gamma$, the antisymmetric tensor formulation would give contributions starting at $O(p^4)$ while QCD requires an explicit $O(p^3)$ term given by the conventional vector formulation. As already realized in [20], using the conventional vector formulation,

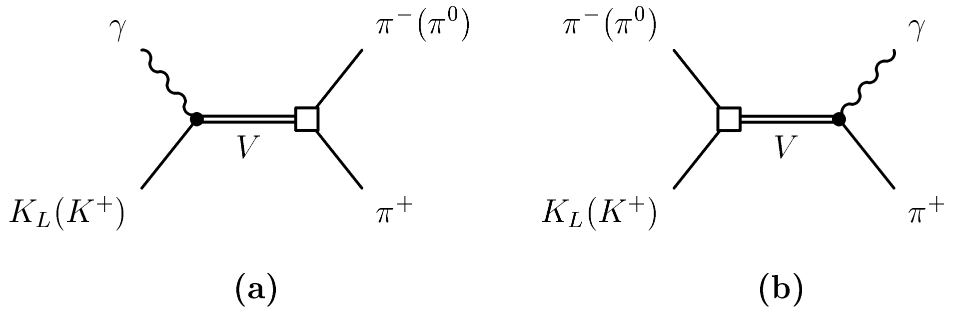

Figure 2: Diagrams contributing to the direct VMD magnetic amplitude of $K_L \to \pi^+\pi^-\gamma$ or $K^+ \to \pi^+\pi^0\gamma$. The empty box denotes the vertex generated by eq. (3.3), and the black circle denotes the strong/electromagnetic vertex. The crossed diagram $\pi^+ \leftrightarrow \pi^-(\pi^0)$ should be considered in (b).

there are $O(p^4)$ VMD contributions generated by the following operators through the direct weak transition (see figure 2):

$$\mathcal{L}_R^{O(p)} = G_8 F^4 \left[\omega_1^R \langle \Delta \{R_\mu, u^\mu\}\rangle + \omega_2^R \langle \Delta u_\mu \rangle \langle R^\mu \rangle\right], \tag{3.3}$$

where $R = V, A$, denoting the vector and axial-vector resonances, respectively. In the factorization, the couplings ω_i^R are

$$\omega_1^R = -\omega_2^R = \sqrt{2}\, \frac{m_R^2}{F^2} f_R \eta_R\,, \tag{3.4}$$

with η_R is the factorization parameter. f_V and f_A are the effective couplings in the general strong/electromagnetic lagrangian involving spin-1 resonances (We use the notations in ref. [16]). As shown in refs. [20, 16], the spin-1 resonance contributions to a_i's have been obtained. Indeed, the operators in eq. (3.3) with $R = V$ do generate the structure of the VMD form-factor starting at $O(p^4)$ once the full propagator of the vector resonance is taken into account (the axial-vector only contributes a constant). The use of the full VMD propagator was also suggested in $K \to \pi\gamma^*$ [21].

The next leading order magnetic amplitude of $K \to \pi\pi\gamma$ in χPT is at $O(p^6)$, which contains two parts: local contribution and loop contribution. Although the general local $O(p^6)$ couplings in weak effective lagrangian have not been developed yet, we can be sure that many unknown parameters have to appear, which will make the prediction impossible. One may expect that the local terms could be generated through resonance exchange which are reasonably thought as the most relevant ones, and the large number of unknown couplings could be reduced significantly. But a complete determination of the contributions from resonances including vector, axial-vector, scalar, and pseudoscalar still remains very difficult because we have to face some unknown pseudoscalar-resonance weak couplings which cannot be fixed by experiments. Also, at $O(p^4)$, we know that the couplings a_i's have other contributions than that from the resonances: for instance, there exists the contribution from the

WZW anomaly action, giving $0 < a_i^{\rm an} \leq 1.0$ [14, 22] with $a_i^{\rm an}$ is the unknown parameter. In the K_L case, there is $O(p^6)$ F_1 term, which is sensitive to the octet symmetry breaking [4, 16].

On the other hand, one can reasonably assume that the photon energy dependence of the amplitude is dominated by the vector resonance exchange while the other resonances including axial-vector, scalar and pseudoscalar only generate the constants contributing to the amplitude. Moreover, we have checked the $O(p^6)$ loop would lead to very negligible energy dependent contribution in $K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$. Therefore, a phenomenological description is that, one can express the full magnetic amplitude as non-VMD part (which is the constant but with large uncertainties involved in it) and VMD part (which is energy dependent and could be determined up to one parameter), and use the corresponding experimental decay rate to determine the former part. Here we use the recent observed values of $K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$:

$$Br\Big(K_L \to \pi^+\pi^-\gamma; E_\gamma^* > 20\,\mathrm{MeV}\Big)_{\rm DE} = (3.10 \pm 0.05) \times 10^{-5}\ [17]\,, \tag{3.5}$$

and

$$Br\Big(K^+ \to \pi^+\pi^0\gamma; 55\,\mathrm{MeV} \leq T_c^* \leq 90\,\mathrm{MeV}\Big)_{\rm DE} = (4.7 \pm 0.8) \times 10^{-6}\ [23]\,. \tag{3.6}$$

The VMD part magnetic amplitude of $K_L \to \pi^+\pi^-\gamma$, corresponding to figures 1 and 2, gives

$$M_{\rm VMD}^L = \frac{eG_8 m_K^3}{2\pi^2 F}\tilde{r}\left(\frac{\eta_V + \frac{m_K^2}{m_V^2}(1-2z_3)}{1 - \frac{m_K^2}{m_V^2} + \frac{2m_K^2}{m_V^2}z_3} + \frac{\frac{\eta_V}{2} - \frac{m_K^2}{m_V^2}z_3}{1 - \frac{m_K^2}{m_V^2}z_3}\right), \tag{3.7}$$

with

$$\tilde{r} = \frac{32\sqrt{2}\pi^2 f_V h_V}{3}\,, \tag{3.8}$$

where h_V is the coupling in the general strong/electromagnetic lagrangian involving spin-1 resonances [16]; the η_V part is $O(p^4)$, and the rest is $O(p^6)$.

We would like to give some remarks here:

1. The VMD form factor like eq. (1.1) was firstly suggested by Lin and Valencia [6], and it is phenomenologically successful. However, as already noted by Picciotto [24], theoretically, there exists some inconsistency in that version because the indirect VMD form factor should vanish at $O(p^4)$ but the $O(p^6)$ indirect vector exchange contribution is important in understanding the magnetic transition of $K_L \to \pi^+\pi^-\gamma$ [24, 4]. Here, we have included both direct and indirect VMD contributions to the form factor eq. (3.7) with the former one starting at $O(p^4)$ and the latter one starting at $O(p^6)$, which properly satisfies all the theoretical constraints.

η_V	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
A^L	0.55	0.36	0.17	-0.01	-0.20	-0.38	-0.57	-0.75	-0.94	-1.13
A^+	1.53	1.34	1.16	0.97	0.79	0.60	0.42	0.23	0.05	-0.13

Table 1: The quantities A^L and A^+ extracted from the observed branching ratio eqs. (3.5) and (3.6) for the different η_V.

2. The second term in eq. (3.7), divided by $(1-\frac{m_K^2}{m_V^2}z_3)$, is absent in eq. (1.1). However, theoretically, it is generated by figure 1b (corresponding to z_3 part) and figure 2b (corresponding to η_V part), we have no reason to exclude it. In fact it constructively enhances the slope of z_3, thus affects the rate and the spectrum significantly. Note that, as a good approximation, we have used $z_+ = z_- \simeq z_3/2$ in deriving this term.

3. The ω_2^V term in eq. (3.3) does not contribute to $K_L \to \pi^+\pi^-\gamma$, so we do not need the factorization relation eq. (3.4) in deriving eq. (3.7). This means the present calculation in K_L case is independent of the factorization. It is an almost model-independent prediction.

The non-VMD part can be written as

$$M^L_{\text{non-VMD}} = \frac{eG_8 m_K^3}{2\pi^2 F} A^L\,, \tag{3.9}$$

with

$$A^L = (a_2 + 2a_4)_{\text{non-VMD}} + \text{other contributions}\,. \tag{3.10}$$

The values of A^L are displayed in the second line of table 1 in the range of $0.1 \leq \eta_V \leq 1.0$. We find that the spectrum of the photon energy is not very sensitive to the value of η_V, as shown in figure 3, and they are in agreement with the one generated from eq. (1.1), which gives the best χ^2 fit to the data in ref. [17]. From figure 3, the best value of η_V is about 0.5, which is reasonably consistent with the preferred $\eta_V \simeq 0.3$ obtained in the factorization [19]. The difference is that, some other $O(p^6)$ contributions parametrized using $\eta_{\text{VP}\gamma}$ and η_{VPP} are considered there [19, 16]. Therefore, it seems that the high order contributions could enhance the value of η_V.

Likewise, in $K^+ \to \pi^+\pi^0\gamma$, the VMD (from figures 1 and 2) and non-VMD parts magnetic amplitudes are

$$M^+_{\text{VMD}} = -\frac{eG_8 m_K^3}{4\pi^2 F}\tilde{r}\left(\frac{\eta_V + \frac{m_K^2}{m_V^2}(1-2z_3)}{1-\frac{m_K^2}{m_V^2}+\frac{2m_K^2}{m_V^2}z_3} + \frac{-\frac{\eta_V}{2}+\frac{2m_K^2}{m_V^2}z_+}{1-\frac{2m_K^2}{m_V^2}z_+} + \frac{\eta_V + \frac{2m_K^2}{m_V^2}z_0}{1-\frac{2m_K^2}{m_V^2}z_0}\right), \tag{3.11}$$

and

$$M^+_{\text{non-VMD}} = -\frac{eG_8 m_K^3}{4\pi^2 F} A^+\,, \tag{3.12}$$

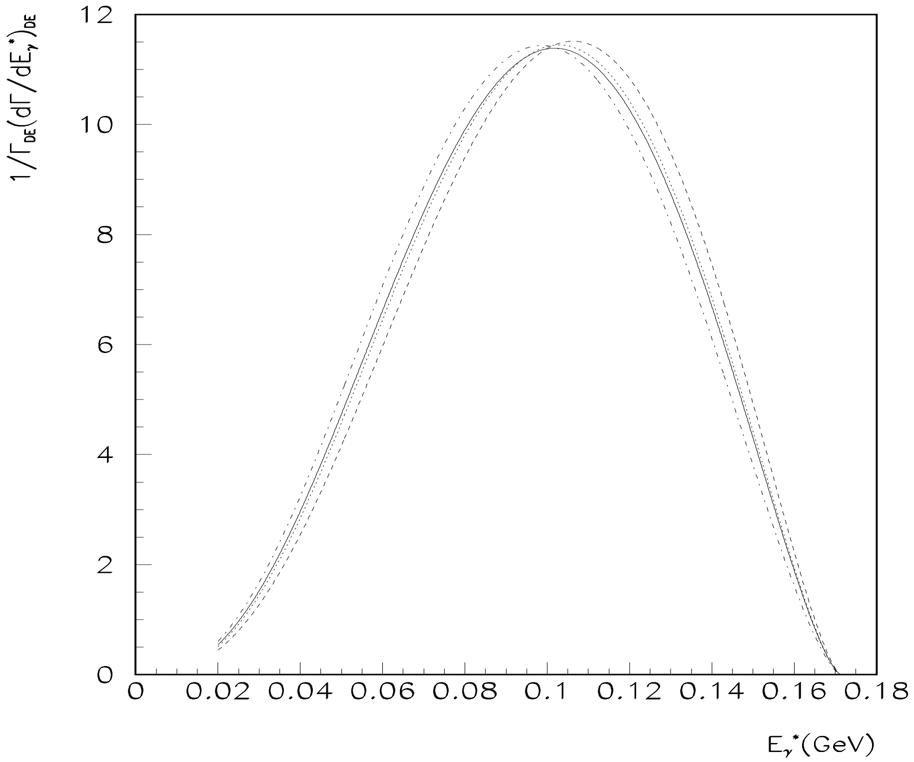

Figure 3: E^*_γ spectrum of the DE $K_L \to \pi^+\pi^-\gamma$ ($E^*_\gamma > 20\,\mathrm{MeV}$). The solid line is from eq. (1.1). The rest are generated from eq. (3.7): the dashed line corresponds to $\eta_V = 0.1$, the dotted line corresponds to $\eta_V = 0.5$, and the dot-dashed line corresponds to $\eta_V = 1.0$.

with

$$A^+ = 2 + 3(2a_3 - a_2)_{\text{non-VMD}} + \text{other contributions}\,. \tag{3.13}$$

We have shown in table 1 the values of A^+ for the different η_V, and plotted the T^*_c normalized to m_K and W spectrum from the DE magnetic amplitude in figures 4 and 5, and the W spectrum from the sum of IB and DE amplitude normalized to the IB spectrum in figure 6. Also,we find that these spectra are not sensitive to the η_V, and the last one can be compared with the corresponding experimental result [23].

4. Conclusions

We have presented a phenomenological description of the magnetic amplitudes of $K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$ beyond the leading order in χPT. The VMD contribution plays an important role in the analysis. We parameterize the VMD part magnetic amplitudes of these two decays in eqs. (3.7) and (3.11), and the non-VMD parts of the amplitudes are estimated by fitting the corresponding observed decay rates. Our phenomenological description is consistent with the factorization prediction $\eta_V \simeq 0.3$.

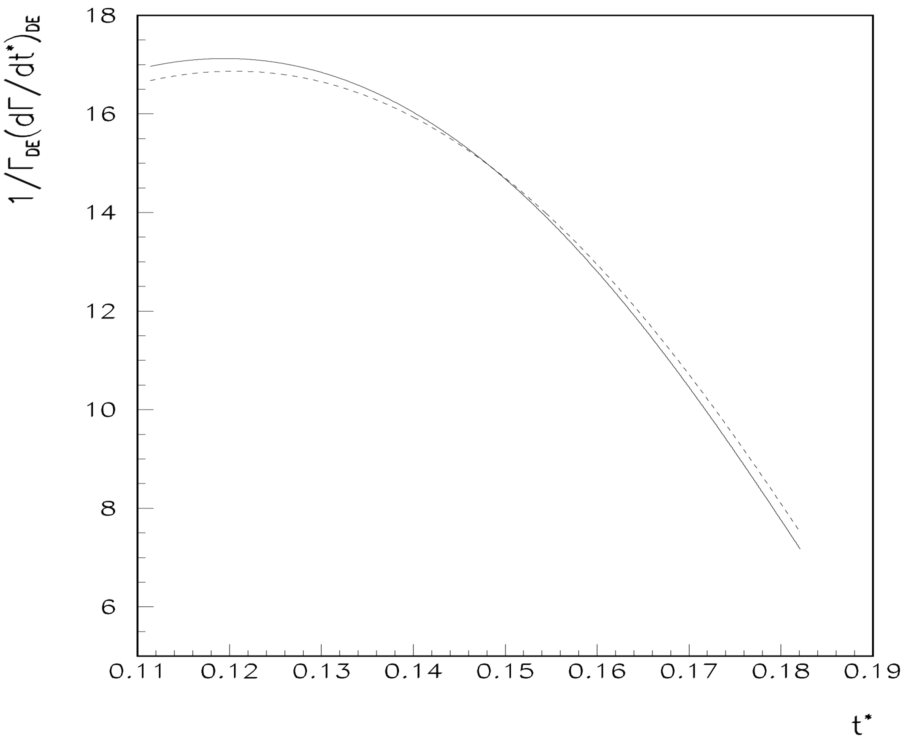

Figure 4: Spectrum in $t^*(=T_c^*/m_K)$ of the DE $K^+ \to \pi^+\pi^0\gamma$ with 55 MeV$\leq T_c^* \leq$90 MeV. The solid line corresponds to $\eta_V = 0.1$. The dashed line corresponds to $\eta_V = 1.0$.

We summarize the analysis as follows.

1. We get the values of A^L and A^+ in the range of $0.1 \leq \eta_V \leq 1.0$ (see table 1). A^L is equals to $(a_2 + 2a_4)_{\text{non-VMD}}$ plus other higher order contributions. We know some other high order contributions, for instance, F_1 term is important and very sensitive to the octet symmetry breaking [4, 16]. So here we cannot expect the conclusive information on $a_2 + 2a_4$ from A^L. The situation in K^+ case seems a little better. After neglecting higher order contributions to A^+, we can get $-0.71 \leq (2a_3 - a_2)_{\text{non-VMD}} \leq -0.1$. From ref. [20] in the factorization, $(2a_3 - a_2)_{\text{axial-vector}} \simeq 0.3\eta_A$ with $0 < \eta_A \leq 1.0$ is the factorization parameter. If we assume the rest contribution to $(2a_3 - a_2)$ is dominated by the one from WZW anomaly action, we find our prediction on $(2a_3 - a_2)^{\text{an}}$ is consistent with the expected $0 < a_i^{\text{an}} \leq 1.0$.

2. Although from our analysis A^L could be positive or negative, the large CP asymmtry B_{CP} [25] in $K_L \to \pi^+\pi^-e^+e^-$ originated from the interference between the magnetic and IB amplitude of $K_L \to \pi^+\pi^-\gamma^*$ is predicted to be always positive and not sensitive to η_V in the present analysis, which is consistent with the measurement [26]. This is not surprising if we carry out Taylor expansion over the form-factor in eq. (3.7) (we assume we can do this expan-

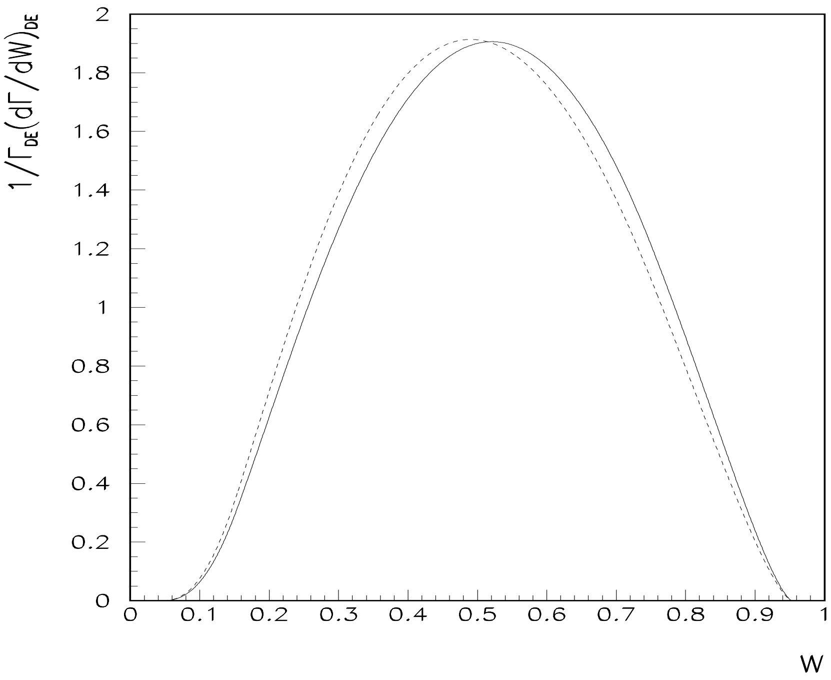

Figure 5: Spectrum in W of the DE $K^+ \to \pi^+\pi^0\gamma$. The solid line corresponds to $\eta_V = 0.1$. The dashed line corresponds to $\eta_V = 1.0$.

sion), and express the total amplitude as

$$M = \frac{eG_8 m_K^3}{2\pi^2 F}\tilde{m}(1 + rz_3 + sz_3^2)\,, \tag{4.1}$$

we will get, in the range of $0.1 \le \eta_V \le 1.0$,

$$1.47 \le \tilde{m} \le 1.75\,, \qquad 2.08 \le -r \le 2.88\,, \qquad 2.50 \le s \le 3.93\,, \tag{4.2}$$

which are comparable with the recent KTeV measurement $r = -2.93 \pm 0.41 \pm 0.34$, $s = 3.31 \pm 1.15 \pm 0.96$ [17], and $|\tilde{m}| = 1.53 \pm 0.25$ in ref. [9] from only linear slope fit.

3. So far, there is no experimental evidence for the energy dependence of the magnetic amplitude in $K^+ \to \pi^+\pi^0\gamma$. But the VMD form-factor obviously indicates this energy dependence. By Taylor expansion of eq. (3.11), we can get the corresponding linear slopes of z_+ and z_0: $0.77 \le -r_+ \le 1.61$, $0.72 \le -r_0 \le 1.17$ in the range of $0.1 \le \eta_V \le 1.0$. These values are not small if this kind of Taylor expansion is valid here. Unfortunately, it is not very easy to measure these quantities experimentally. On the other hand, we find that, z_0 is related to T_c^* through a linear relation eq. (2.6). Therefore, it is expected that a high-precision experimental analysis of the T_c^*/m_K distribution from DE contribution may be able to measure this energy dependence of the amplitude.

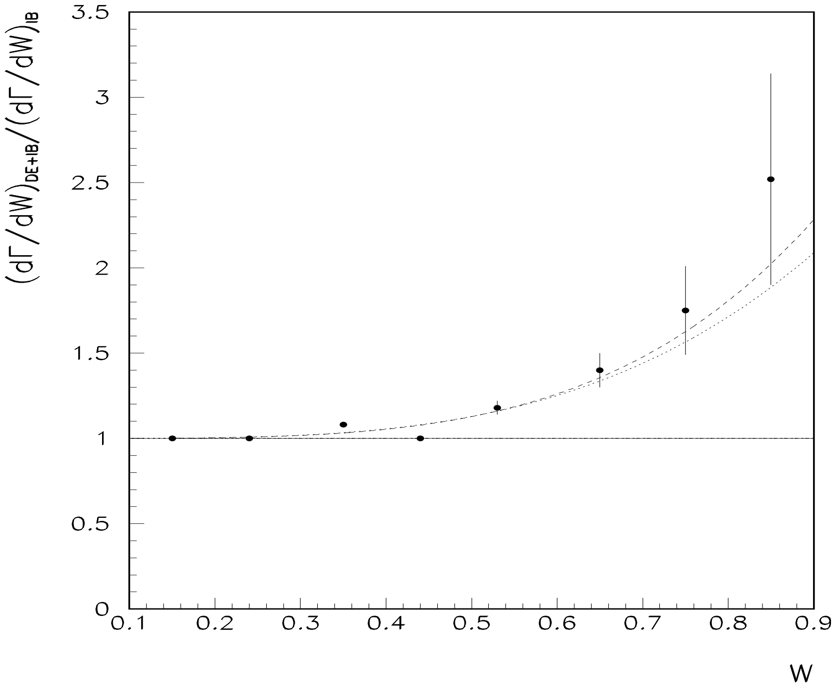

Figure 6: W spectrum normalized to the IB spectrum of $K^+ \to \pi^+\pi^0\gamma$. The dashed line corresponds to $\eta_V = 0.1$. The dotted line corresponds to $\eta_V = 1.0$. The data points are from ref. [23].

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